OGAP Sample Problems: Contexts, Topics and Types (January 2016)

Important: The samples below do not include the full range of problem contexts or problem types.

Ratio - referent explicit:
There are red and blue marbles in a bag. The ratio of red to blue marbles is 1:2. Sue opened the bag and found 12 red marbles. How many blue marbles are in the bag?

Ratio - referent not given, but implied:
There are red and blue marbles in a bag. The ratio of red to blue marbles is 1:2. Sue opened the bag and found 12 red marbles. How many marbles are in the bag altogether?

Rate - Density

<table>
<thead>
<tr>
<th>Township A</th>
<th>Township B</th>
</tr>
</thead>
<tbody>
<tr>
<td>42 square miles</td>
<td>44 square miles</td>
</tr>
<tr>
<td>51 raccoons</td>
<td>45 raccoons</td>
</tr>
</tbody>
</table>

Karl says that Township A has more raccoons per square mile. Josh says Township B has more raccoons per square mile. Who is right? Justify your answer.

Rate - Constant rate of speed
A train is moving at a constant rate of speed. It has traveled 230 miles in 5 hours. At this rate how many miles will the train travel in 9 hours?

Rate - Buy/consume - Unit Rate
A 20-ounce box of Toasty Oats costs $2.98. How much does Toasty Oats cost per ounce?

Rate Comparison
A 20-ounce box of Toasty Oats costs $2.98. A 16-ounce box of Toasty Oats costs $2.48. Which box of Toasty Oats is a better buy?

Similarity
The dimensions of 4 rectangles are given below. Which 2 rectangles are similar? Show your work
a) 2 inches x 3 inches  b) 4 inches x 5 inches  c) 6 inches x 9 inches  d) 6 inches x 10 inches

Scale Factor
Richard enlarged a photo using a scale factor of 1.8. If the dimensions of the original photo was 5 inches by 7 inches, what are the dimensions of the enlarged photo?

Missing Value
Bob’s shower uses 12 gallons of water in 3 minutes. How many gallons of water does Bob use if he takes a shower in 12 minutes?

Measurement Conversions
How many feet of material is 4.5 meters?

Qualitative
Nick ran exactly the same number of laps around the school track yesterday as he ran today. If it took him less time to run the laps today than yesterday, his running speed today would be:

a) faster  b) slower  c) exactly the same  d) Not enough information. Explain your choice.

Non-proportional
Kim and Bob were running equally fast around a track. Kim started first. When she had run 9 laps, Bob had run 3 laps. When Bob had run 15 laps, how many laps had Kim run?

Percent
Percent of problems (all three types: percent unknown, whole unknown, and part unknown)
Simple interest; tax; tips; discount; markups; commissions
Percent increase and decrease
Percent error

OGAP Proportionality Framework

PROBLEM STRUCTURES

Contexts and Topics
Types of Problems
Multiplicative Relationships
Ratio Referent
Ratios
Rates (density)
Rates (D=RT)
Rates (buy/consume)
Rate concentrations
Measurement conversions
Similarity
Scale
Probability
Percents
Slope
Sampling
Both integral
Both non-integral
Some non-integral
All whole numbers
Rational numbers
Part: whole
Part: part
Coordinate Graph
Table
Model
Expression
Equation

OGAP frameworks are based on mathematics education research on how students learn specific mathematics concepts, errors students make, and pre-conceptions or misconceptions that may interfere with learning new concepts or solving related problems.

There are three major elements to an OGAP Framework that should be considered when analyzing student work or making instructional decisions:

1) Problem structures
2) Evidence in student work along a progression
3) Evidence of underlying issues or errors

This page identifies contexts, topics, and problem structures for ratio and proportion problems. The centerfold is a learning progression to help teachers classify evidence in student work, including classroom discussions, and make instructional decisions. Sample OGAP questions reflecting different contexts, topics, and types of problems are shown on Page 4.

For students to become strong in their application of ratios, rates, and proportional relationships they must interact with a range of problem contexts and problem structures. Beginning in grade 4 the CCSSM engages students in multiplicative change problems which set the foundation for proportional thinking. In grade 6 students focus on understanding ratios and unit rates and applying these concepts in problems involving unit rate, constant rate of speed, measure conversions, and percent. In grade 7 this work is extended to proportional relationships and solving multi-step proportion problems, percent problems, scale problems, and developing probability. In grade 8 students study the connections between proportional relationships, graphs, and linear equations including understanding of similarity.

Consistent with the CCSSM the OGAP Frameworks on the centerfold shows a progression from using models, building up and down, and ratio tables to efficient and generalized strategies identified in Proportional Strategies in the OGAP Proportionality Framework.

As students interact with new concepts, new problem contexts, new structures, and more complex problem solving situations the strategies that they use may move back and forth between using proportional, transitional, and non-proportional strategies. This is important evidence to use for instructional decision making. For example, a student may consistently solve problems using unit rate as a proportional strategy. However, when given a density comparison problem students may revert to a non-proportional strategy.
**OGAP Proportionality Progression (January 2016)**

**Finds and applies unit rate**
Bob's shower uses 18 gallons of water in 3 minutes. How many gallons of water does Bob use if he takes a shower in 13 minutes?

\[
\frac{18 \text{ gal}}{3 \text{ min}} \times \frac{13 \text{ min}}{1} = \frac{234 \text{ gal}}{3} = 78 \text{ gal}
\]

**Applies multiplicative relationship**
Bob's shower uses 14 gallons of water in 3 minutes. How many gallons of water does Bob use if he takes a shower in 8 minutes?

\[
\frac{14 \text{ gal}}{3 \text{ min}} \times \frac{8 \text{ min}}{1} = \frac{112 \text{ gal}}{3} = 37 \frac{1}{3} \text{ gal}
\]

**Uses \( y = \frac{kx}{\text{rate}} \) (either symbolic or graphical representation moving between graphs, tables, and equations)**
Bob's shower uses 14 gallons of water in 3 minutes. How many gallons of water does Bob use if he takes a shower in 12 minutes?

\[
\frac{14 \text{ gal}}{3 \text{ min}} \times \frac{12 \text{ min}}{1} = \frac{168 \text{ gal}}{3} = 56 \text{ gal}
\]

**Sets up a proportion and uses cross products**
Bob's shower uses 18 gallons of water in 3 minutes. How many gallons of water does Bob use if he takes a shower in 13 minutes?

\[
\frac{18 \text{ gal}}{3 \text{ min}} \times \frac{13 \text{ min}}{1} = \frac{234 \text{ gal}}{3} = 78 \text{ gal}
\]

**Compares simplified fractions, rates, and ratios**
The dimensions of 4 rectangles are given below. Which 2 rectangles are similar?

A) 2" X 8"  
B) 4" X 10"  
C) 6" X 12"  
D) 6" X 15"

**Applies the correct ratio referent in a ratio problem**
There are red and blue marbles in a bag. The ratio of red to blue marbles is 1:2. Sue opens the bag and found 12 red marbles. How many marbles are in the bag altogether?

\[\text{Red marbles: Blue marbles = 1:2} \]
\[12 \text{ red marbles} \rightarrow 24 \text{ blue marbles} \]
\[12 + 24 = 36 \text{ marbles} \]

**Uses double number line**
Paul's dog eats 20 pounds of food in 30 days. How long will it take Paul’s dog to eat 45 pounds of dog food?

\[
\frac{20 \text{ lbs}}{30 \text{ days}} \times \frac{30 \text{ days}}{1} = \frac{600 \text{ lbs}}{1} = 20 \text{ days}
\]

**Uses a proportional strategy for part of the problem**
Carrie is packing apples for an orchard’s mail order business. It takes 3 boxes to pack 2 bushels of apples. How many boxes will she need to pack 7 bushels of apples?

\[\text{3 boxes} \rightarrow 2 \text{ bushels} \]
\[7 \text{ bushels} = \frac{7 \times 3}{2} = 10 \frac{1}{2} \text{ boxes} \]

**Uses ratio table with skip counting or repeated addition.**
Bob's shower uses 18 gallons of water in 3 minutes. How many gallons of water does Bob use if he takes a shower in 13 minutes?

<table>
<thead>
<tr>
<th>Minutes</th>
<th>Gallons</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>6</td>
<td>36</td>
</tr>
<tr>
<td>9</td>
<td>54</td>
</tr>
<tr>
<td>12</td>
<td>72</td>
</tr>
<tr>
<td>13</td>
<td>78</td>
</tr>
</tbody>
</table>

**Builds up or down**
Bob's shower uses 18 gallons of water in 3 minutes. How many gallons of water does Bob use if he takes a shower in 12 minutes?

\[
\frac{18 \text{ gal}}{3 \text{ min}} \times \frac{12 \text{ min}}{1} = \frac{216 \text{ gal}}{3} = 72 \text{ gal}
\]

**Uses additive difference, not multiplication relationship**
Bob's shower uses 14 gallons of water in 3 minutes. How many gallons of water does Bob use if he takes a shower in 8 minutes?

\[
\frac{14 \text{ gal}}{3 \text{ min}} \times \frac{8 \text{ min}}{1} = \frac{112 \text{ gal}}{3} = 37 \frac{1}{3} \text{ gal}
\]

**Compares numbers, not ratios or rates**
Karl says that Town A has more raccoons per square mile. Josh says that Town B has more raccoons per square mile. Who is right? Justify your answer.

**Use of visual models, ratio and rate tables, equivalence, contexts, and unit rates**
The strategies students use move back and forth across the levels as they learn new concepts and/or interact with new problems structures and contexts.

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